Chiral Spin Liquid from Dzyaloshinskii-Moriya Interactions

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Introduction to spin liquids

- Spin Liquids
  - Retains rotational and translational symmetries down to the lowest temperatures
  - No magnetic long range order
  - Chiral spin liquids (chiral order present)
    - Scalar chiral spin liquid (sCSL) breaks time-reversal symmetry

    → $\langle S_i \cdot S_j \times S_k \rangle \neq 0$
    - Vector chiral spin liquid (vCSL) breaks inversion symmetry

    → $\langle z \cdot S_i \times S_j \rangle \neq 0$
  - Chirality arises due to frustration from geometry or competing interactions
    - Triangular, kagome, pyrochlore, J_1–J_2, *etc.*

Studies on spin liquids

- Major research topic for past couple of decades
  - Search for spin liquid states usually starts with Heisenberg Hamiltonian on various frustrated lattices
    - Kalmeyer & Laughlin, PRL **59**, 2095 (87)
    - Wen, Wilczek, & Zee, PRB **39**, 11413 (89)
  - Systems with larger atomic numbers and complex lattices with less symmetry becoming more important
    - → Spotlight on Dzyaloshinskii–Moriya (DM) interaction effects
    - **$Na_3Ir_3O_8$ (Hyper-Kagome spin liquid)**
      Okamato, Nohara, Aruga-Katori, & Takagi, PRL **99**,137207 (07)
      Chen & Balents, PRB **78**, 094403 (08)
      Zhou, Lee, Ng, & Zhang, PRL **101**, 197201 (08)
  - What effect does DM have on spin liquid physics?

Method of approach

- J_1–J_2 spin model without DM

1. Begin with fermionic mean field theory (fMFT) on the J_1-J_2 Heisenberg Hamiltonian
2. Map out phase diagram w.r.t. J_2/J_1 (degree of frustration)
3. Compare the energies of the mean field states using variational Monte Carlo (VMC)
4. What are the (low-lying) excitations?
5. Order parameter (chirality: scalar or vector?)
6. Add DM interactions and repeat steps 1. through 5.

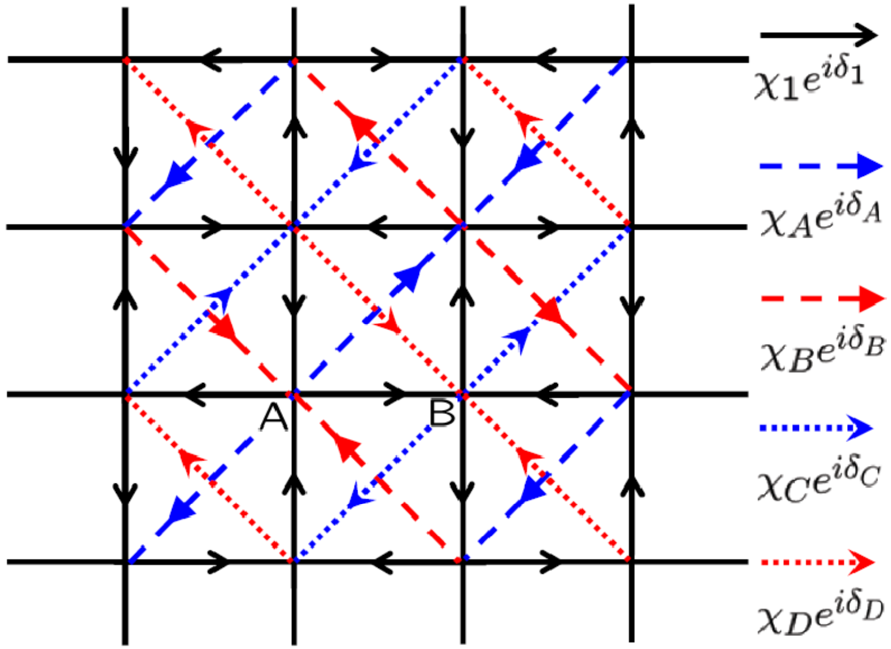
+ DM ⇨ New Phase?

Fermionic mean field theory (fMFT)

- J_1–J_2 spin model on a square lattice

$$H = J_1 \sum_{\langle ij \rangle} S_i \cdot S_j + J_2 \sum_{\langle ik \rangle} S_i \cdot S_k$$

- Constraint : no charge fluctuations, n_i=1
- Rewrite spins as fermionic bilinears
- Define $\chi_{ij,\sigma}$: hopping between sites
- Apply mean field theory

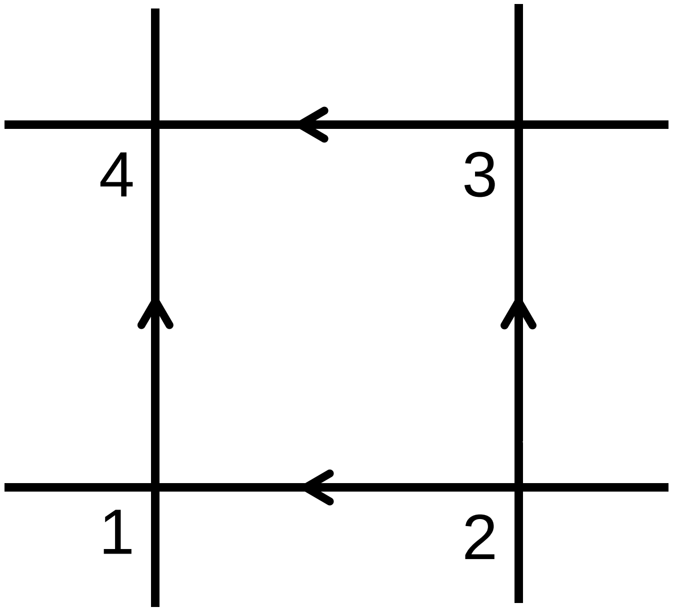


$$S_i = \frac{1}{2} f^{\dagger}_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \qquad \chi_{ij,\sigma} = f^{+}_{i\sigma} f_{j\sigma}$$

⇩

$$H = -J_1 \sum_{\langle ij,\sigma \rangle} \langle \chi_{ij} \rangle f^{+}_{j,\sigma} f_{i,\sigma} - J_2 \sum_{\langle ik,\sigma \rangle} \langle \chi_{ik} \rangle f^{+}_{k,\sigma} f_{i,\sigma} + h.c.$$

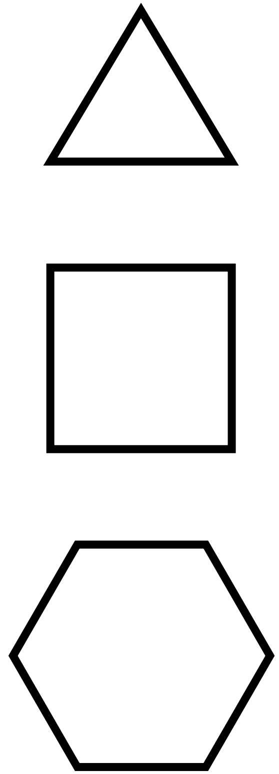
Flux States – Rokshar’s Rule



$$\prod \chi_{ij} = \chi_{12}\chi_{23}\chi_{34}\chi_{41} = \chi e^{i\Phi}$$

Φ : (physical) flux

Rokhsar PRL 65, 1506 (1990)

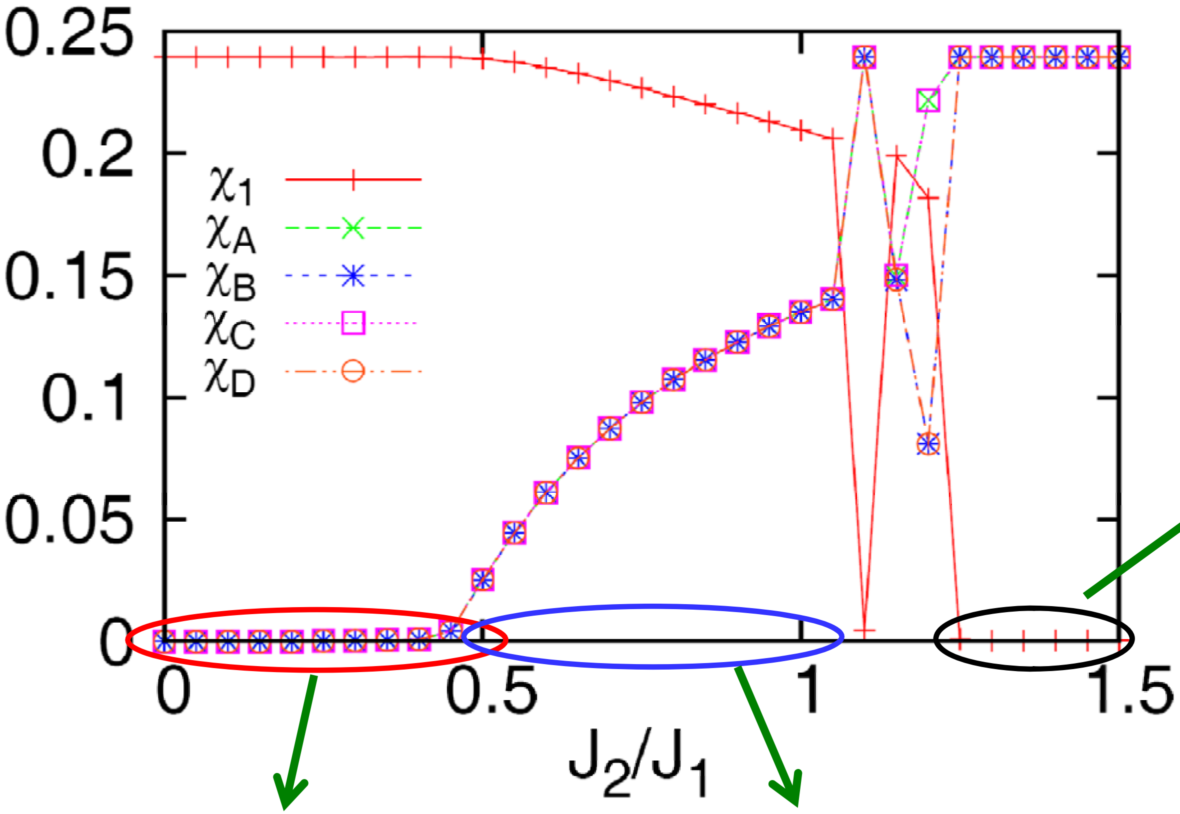


$\Phi = \frac{\pi}{2}$

$\Phi = \pi$

$\Phi = 0$

Phase diagram of $J_1 - J_2$ without DM



See also :
Wen, Wilczek, & Zee
PRB 39, 11413 (89)

Bipartite π−flux

$\chi_A = \chi_B = \chi_C = \chi_D \neq 0$,
$\chi_1 = 0$

π−flux

$\chi_A = \chi_B = \chi_C = \chi_D = 0$,
$\chi_1 \neq 0$, $\delta_1 = \pi/4$

(s)CSL

$\chi_A = \chi_B = \chi_C = \chi_D \neq 0$,
$\delta_A = \delta_D = 0$, $\delta_B = \delta_C = \pi$,
$\delta_1 = \pi/4$

π–flux phases

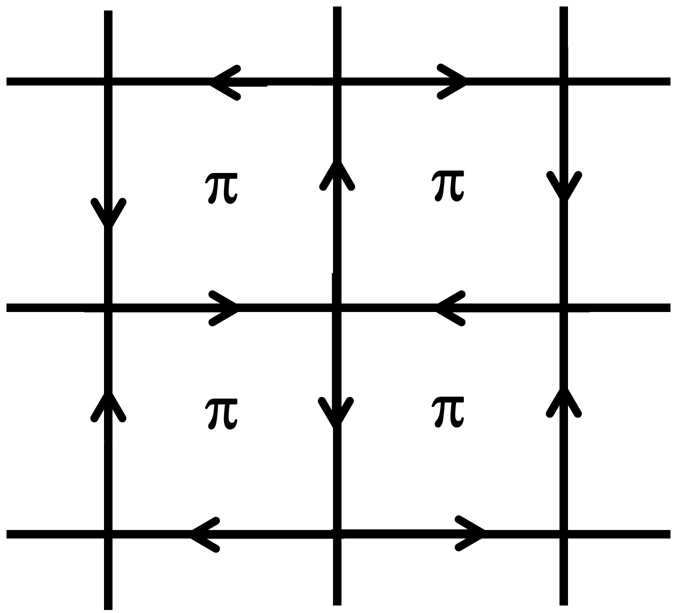
π–flux

$\chi_A=\chi_B=\chi_C=\chi_D=0$,
$\chi_1\neq 0$, $\delta_1=\pi/4$

$\longrightarrow$ $\chi_1 e^{i\delta_1}$

Bipartite π–flux

$\chi_A=\chi_B=\chi_C=\chi_D\neq 0$,
$\chi_1=0$



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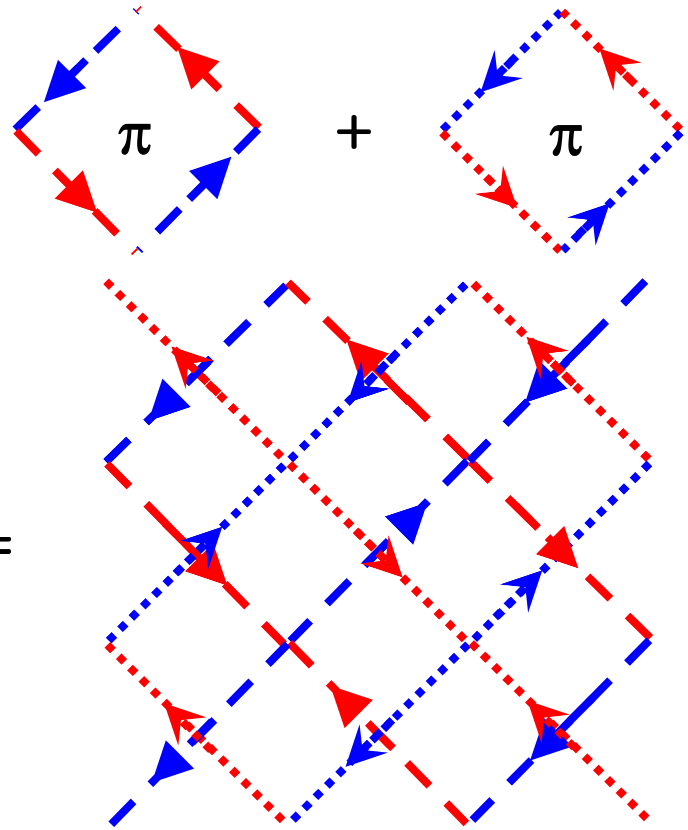


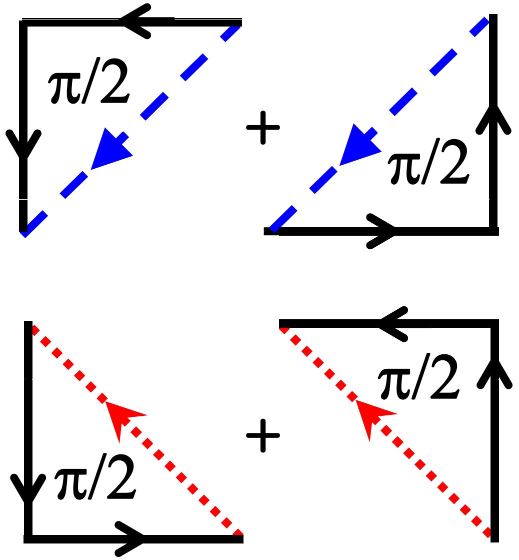
Chiral spin liquid phase

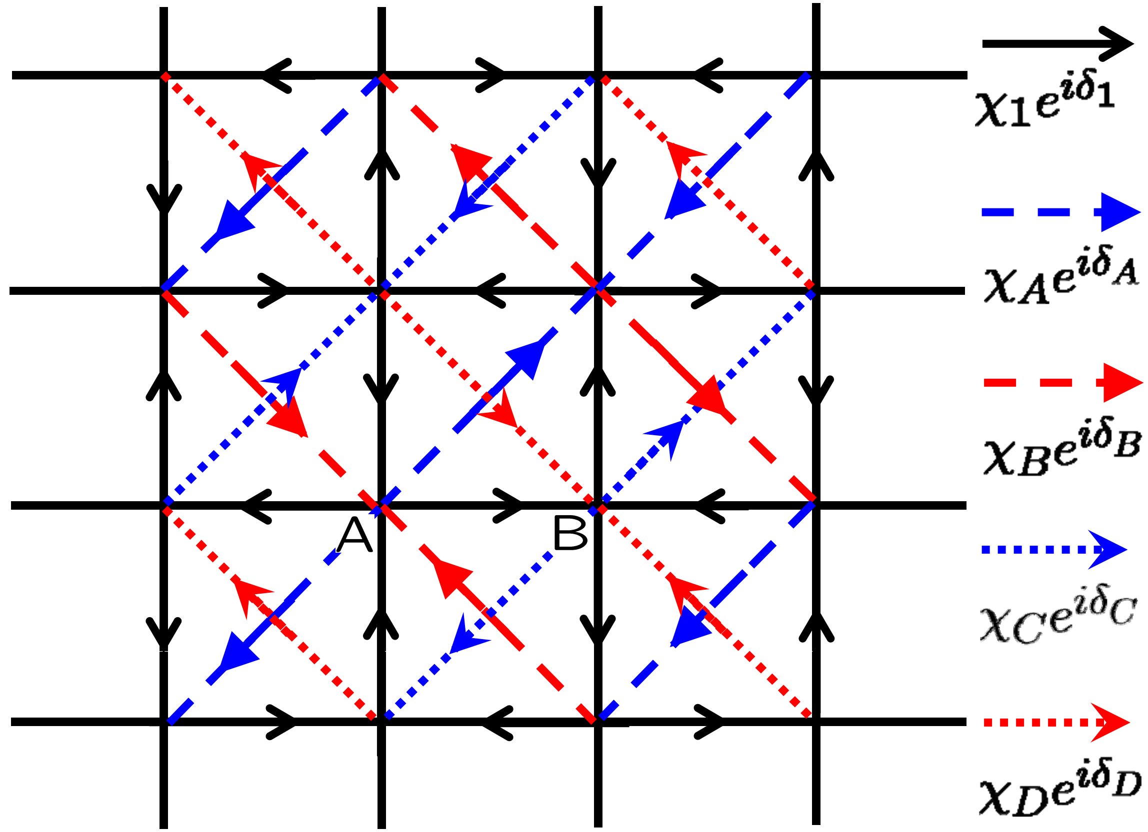
Chiral Spin Liquid

$\chi_A=\chi_B=\chi_C=\chi_D\neq 0,$

$\delta_A=\delta_D=0,\ \delta_B=\delta_C=\pi,$

$\delta_1=\pi/4$

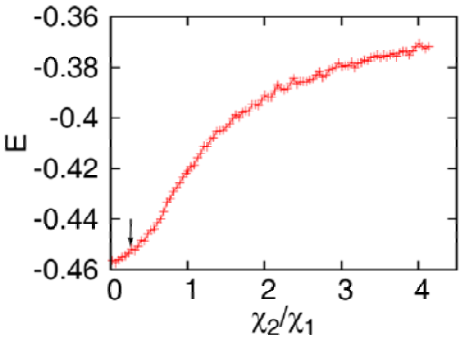




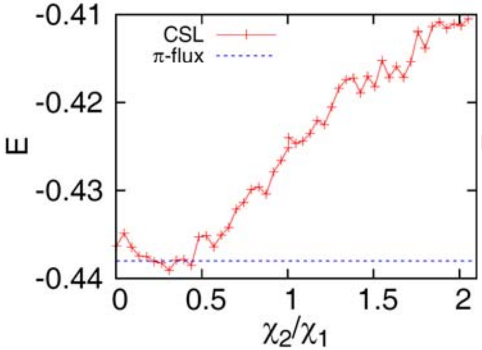
Both up and down spin species "feels" a flux of $\pi/2$ for each triangle

Variational Monte Carlo (VMC) results

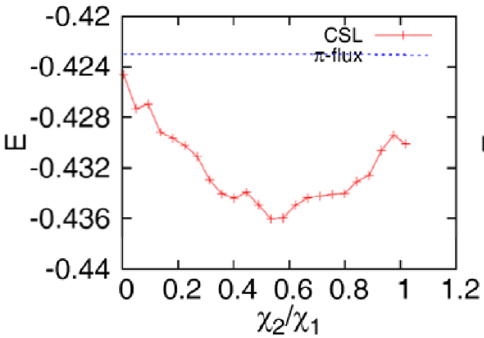
J_2/J_1=0.6



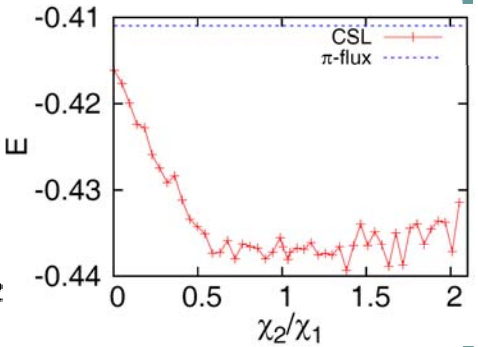
J_2/J_1=0.7

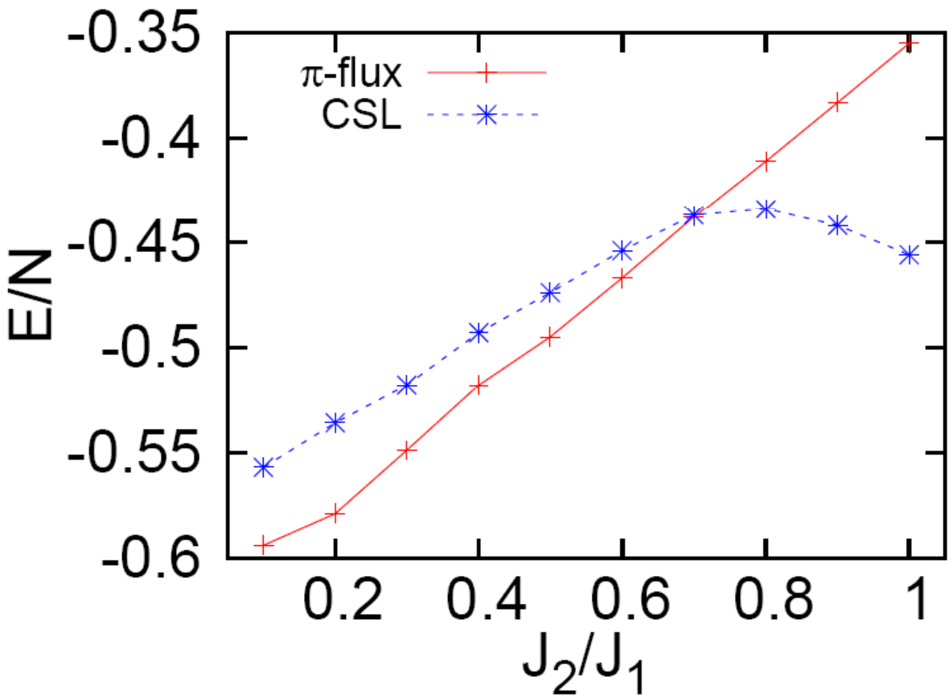


J_2/J_1=0.75



J_2/J_1=0.8





J_2/J_1	π-flux	CSL	χ_2/χ_1
0.70	-0.438	-0.439	0.31
0.75	-0.423	-0.436	0.53
0.80	-0.411	-0.438	1.03*

Heisenberg + DM (HDM model)

- H_{HDM} model has the following form

$$H_{HDM} = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle ik \rangle} \mathbf{S}_i \cdot \mathbf{S}_k + K_1 \sum_{\langle ij \rangle} \mathbf{D}_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j + K_2 \sum_{\langle ik \rangle} \mathbf{D}_{ik} \cdot \mathbf{S}_i \times \mathbf{S}_k$$

$$\mathbf{S}_i \cdot \mathbf{S}_j = -\frac{1}{2}\left(\sum_\sigma \hat{\chi}_{ij,\sigma}\right)\left(\sum_\sigma \hat{\chi}_{ji,\sigma}\right) \qquad \text{where } n_i = \sum_\sigma f^+_{i,\sigma} f_{i,\sigma} = 1$$

$$\mathbf{S}_i \times \mathbf{S}_j \cdot \hat{z} = -\frac{i}{2}\sum_\sigma \sigma \hat{\chi}_{ij,\sigma}\hat{\chi}_{ji,\bar{\sigma}} \qquad \hat{\chi}_{ij,\sigma} = \sum_\sigma f^+_{i,\sigma} f_{j,\sigma}$$

- One can write down a general spin Hamiltonian

$$c_F\left(S_i^\perp \cdot S_j^\perp\right) + s_F\left(S_i \times S_j\right) \cdot \hat{z} = -\frac{1}{2}\sum_\sigma e^{iF\sigma}\chi_{ij,\sigma}\chi_{ji,\sigma}$$

$$\text{where } \tan F = \frac{K_1}{J_1} = \frac{K_2}{J_2}, \quad \mathbf{D}_{ij} = \mathbf{D}_{ik} = \hat{z}$$

Heisenberg + DM (HDM model)

$$H_{\mathrm{HDM}} = \sum_i \sum_{\substack{j=i+\hat{x},\\ i+\hat{y}}} \left(S_i^z S_j^z + \frac{1}{2} e^{iF} S_i^+ S_j^- + \frac{1}{2} e^{-iF} S_i^- S_j^+ \right)$$

$$+ \frac{\kappa}{2} \sum_i \sum_{\substack{k=i+\hat{x}+\hat{y},\\ i-\hat{x}+\hat{y}}} \left(S_i^z S_k^z + \frac{1}{2} e^{iF} S_i^+ S_k^- + \frac{1}{2} e^{-iF} S_i^- S_k^+ \right)$$

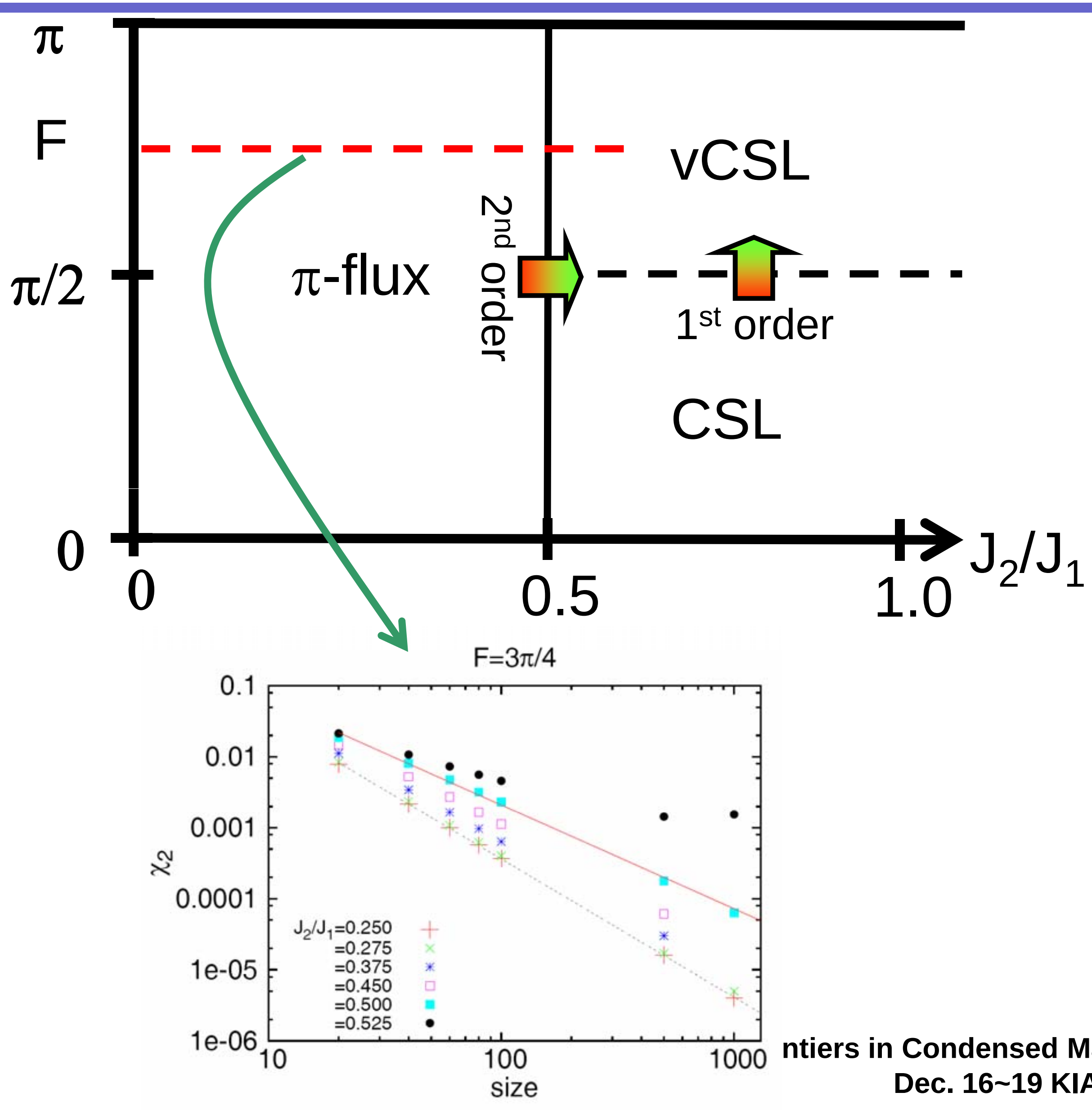
$$\text{where } \kappa = \frac{2J_2}{J_1} = \frac{2K_2}{K_1}, \quad \tan F = \frac{K_1}{J_1} = \frac{K_2}{J_2}$$

- In a fermionic representation

$$H_{\mathrm{HDM}} = \frac{1}{2} \sum_{i,\sigma} \sum_{\substack{j=i+\hat{x},\\ i+\hat{y}}} \left(\hat{\chi}_{ji,\sigma} \hat{\chi}_{ij,\sigma} + e^{iF\sigma} \hat{\chi}_{ji,\bar{\sigma}} \hat{\chi}_{ij,\sigma} \right)$$

$$+ \frac{\kappa}{4} \sum_{i,\sigma} \sum_{\substack{k=i+\hat{x}+\hat{y},\\ i-\hat{x}+\hat{y}}} \left(\hat{\chi}_{ki,\sigma} \hat{\chi}_{ik,\sigma} + e^{iF\sigma} \hat{\chi}_{ki,\bar{\sigma}} \hat{\chi}_{ik,\sigma} \right)$$

Mean-field Phase diagram of H_{HDM}



Vector chiral spin liquid (vCSL) phase

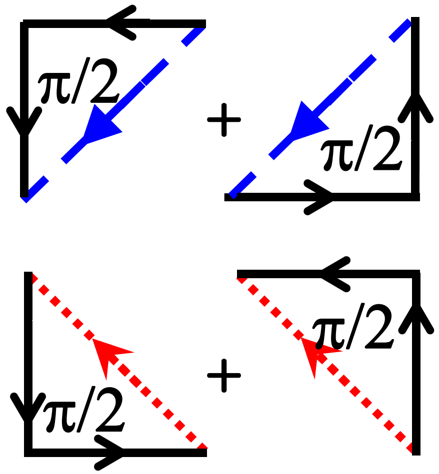
vCSL

$\delta_1=\pi/4,\ \chi_2\neq 0,$

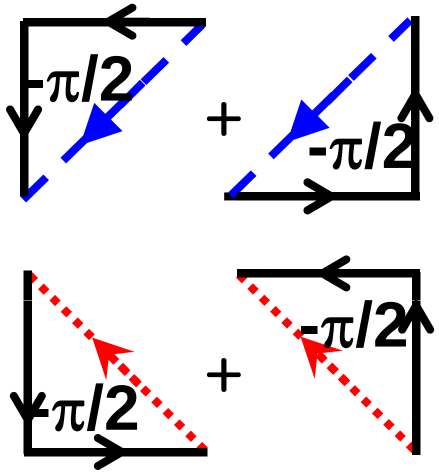
$\delta_{A,\sigma}=\delta_{B,\bar{\sigma}}=\delta_{C,\bar{\sigma}}=\delta_{D,\sigma}=\pi,$

$\delta_{A,\bar{\sigma}}=\delta_{B,\sigma}=\delta_{C,\sigma}=\delta_{D,\bar{\sigma}}=0,$

Up spins :



Down spins :



up spins : flux of $\pi/2$ per triangle

down spins : flux of $-\pi/2$ per triangle

Why vCSL ? (In terms of fermions)

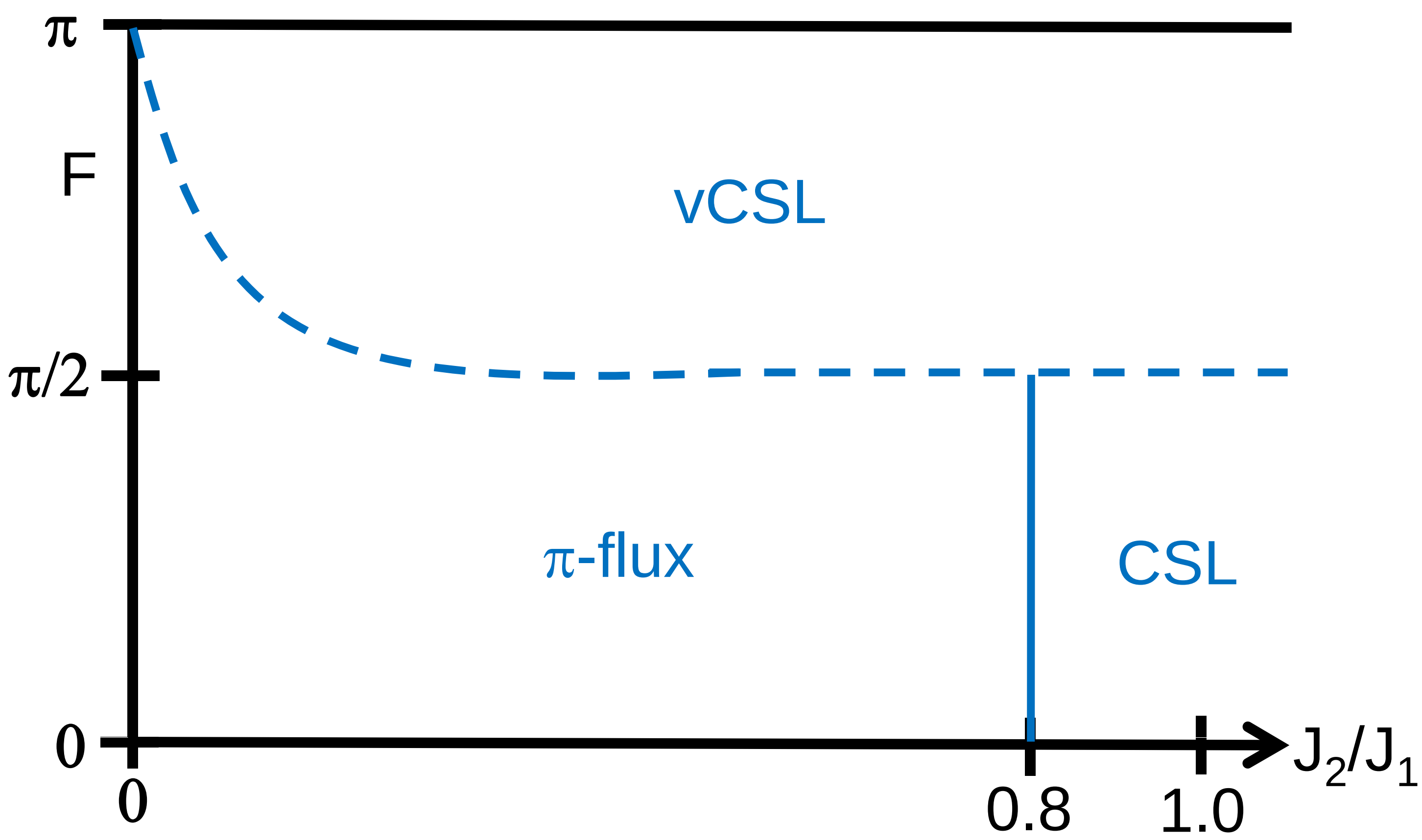
- For the CSL,

$$\langle \mathbf{S}_1 \cdot \mathbf{S}_2 \times \mathbf{S}_3 \rangle = \frac{1}{4i} \langle \left(\sum_\sigma \chi_{13,\sigma} \right) \left(\sum_\sigma \chi_{32,\sigma} \right) \left(\sum_\sigma \chi_{21,\sigma} \right) - h.c. \rangle \neq 0$$

- For the vCSL (for half-filling),

$$\langle (S_2 \times S_3)^z + (S_3 \times S_1)^z + (S_1 \times S_2)^z \rangle$$
$$= \frac{1}{2i} \langle (\sum_\sigma \sigma \chi_{13\sigma})(\sum_\sigma \sigma \chi_{32\sigma})(\sum_\sigma \sigma \chi_{21\sigma}) - h.c. \rangle \neq 0$$

VMC Phase Diagram



Conclusions & future work

- The introduction of DM leads to a new phase, a vector chiral spin phase.
- vCSL phase can be reached from a CSL phase by increasing the DM strength past a certain critical value (first order phase transition).
- In the vCSL phase, the transverse spin Hall conductivity is no longer zero.
- Ground state of H_{HDM} has been established. What about spinon excitations?
- Through VMC, can calculate the vector chirality of vCSL phase. Is it really non-zero?