Hierarchy of Spin and Valley Symmetry Breaking in Quantum Hall Single Layer Graphene

arXiv: 0910.1388

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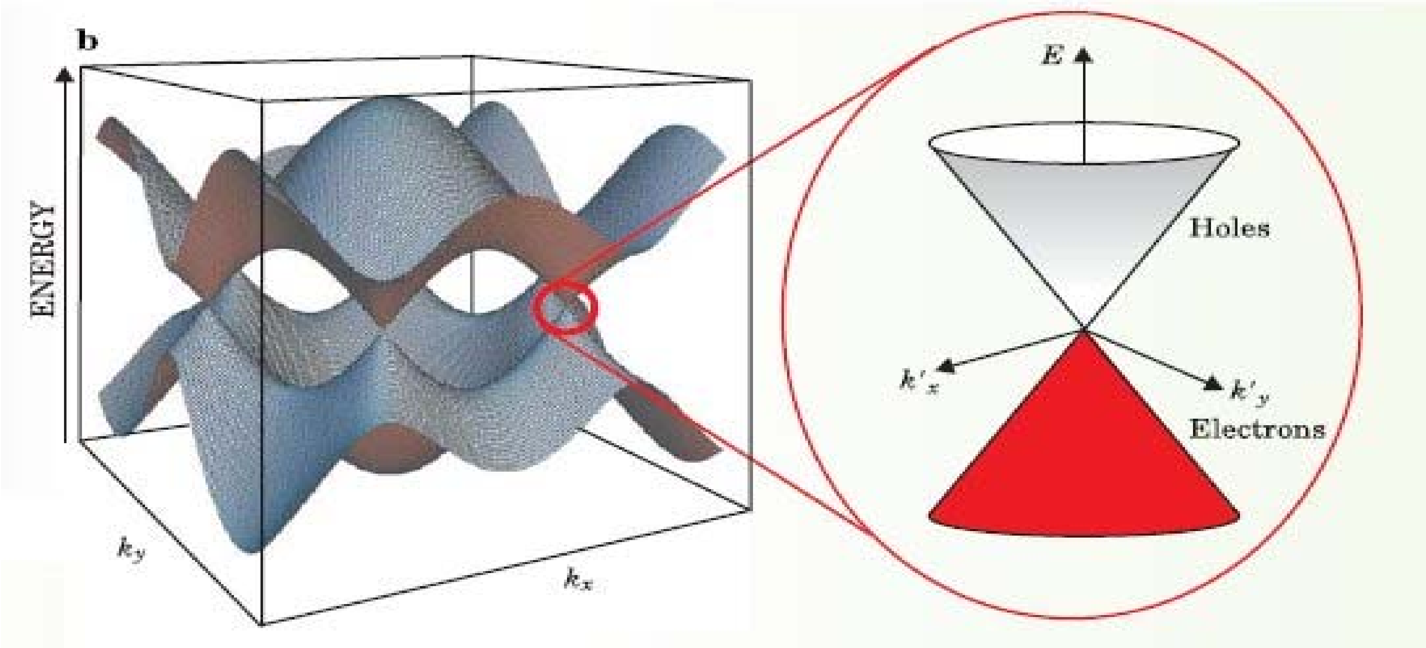
Graphene

• Semi-metal on a honeycomb lattice

• Electron and hole bands touch at two inequivalent points in BZ

K and **K'=-K** (valley)

• Low energy theory is given by Dirac Hamiltonian with energy dispersion relation $E_k = \pm v_F \hbar |\mathbf{k}|$



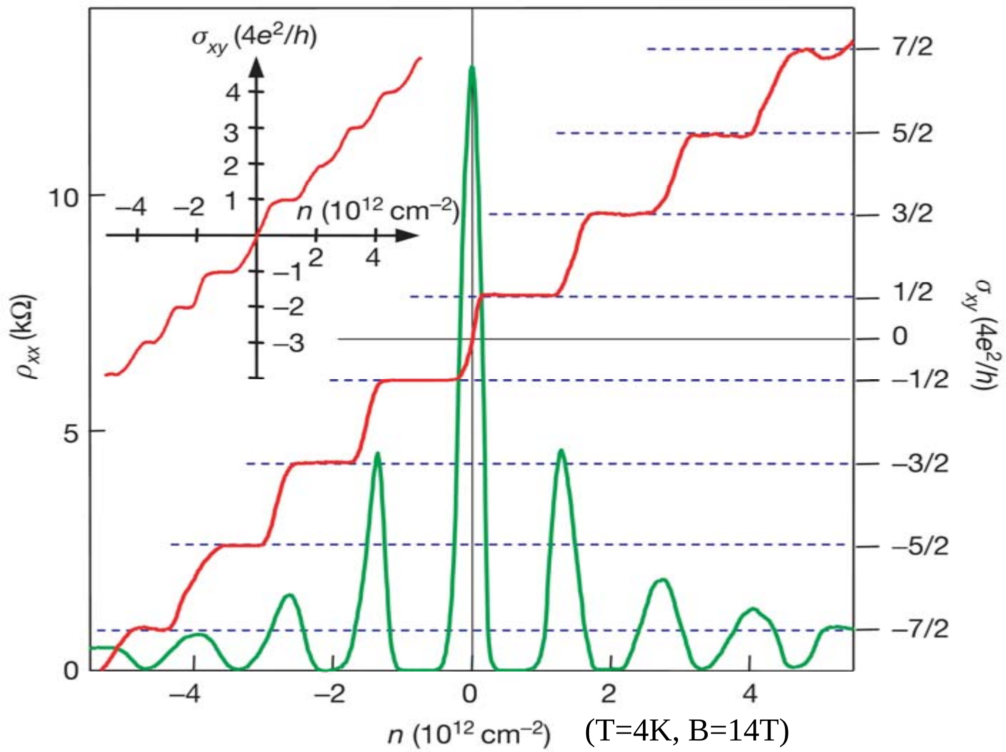
The six corners: Dirac points; holes&electrons: Dirac fermions

Quantum Hall Effect in Graphene

- **Energy spectrum in a perpendicular magnetic field *B***

$$\varepsilon_n = \pm v_F \sqrt{2e\hbar B n}$$

- **Hall conductivity and longitudinal resistivity**



$$\sigma_{xy} = \nu \frac{e^2}{h}\ ,\ \ \nu = \pm 4 \cdot \left(n + \frac{1}{2}\right)$$

Factor 4: spin & valley degeneracy

Factor 1/2: Berry phase due to the valley precession

n : Landau level index

ν : filling factor

K.S.Novoselov,et.al, Nature 438,197(2005)

Y. Zhang,et.al, Nature 438, 201(2005)

"Only problem" in graphene QHE

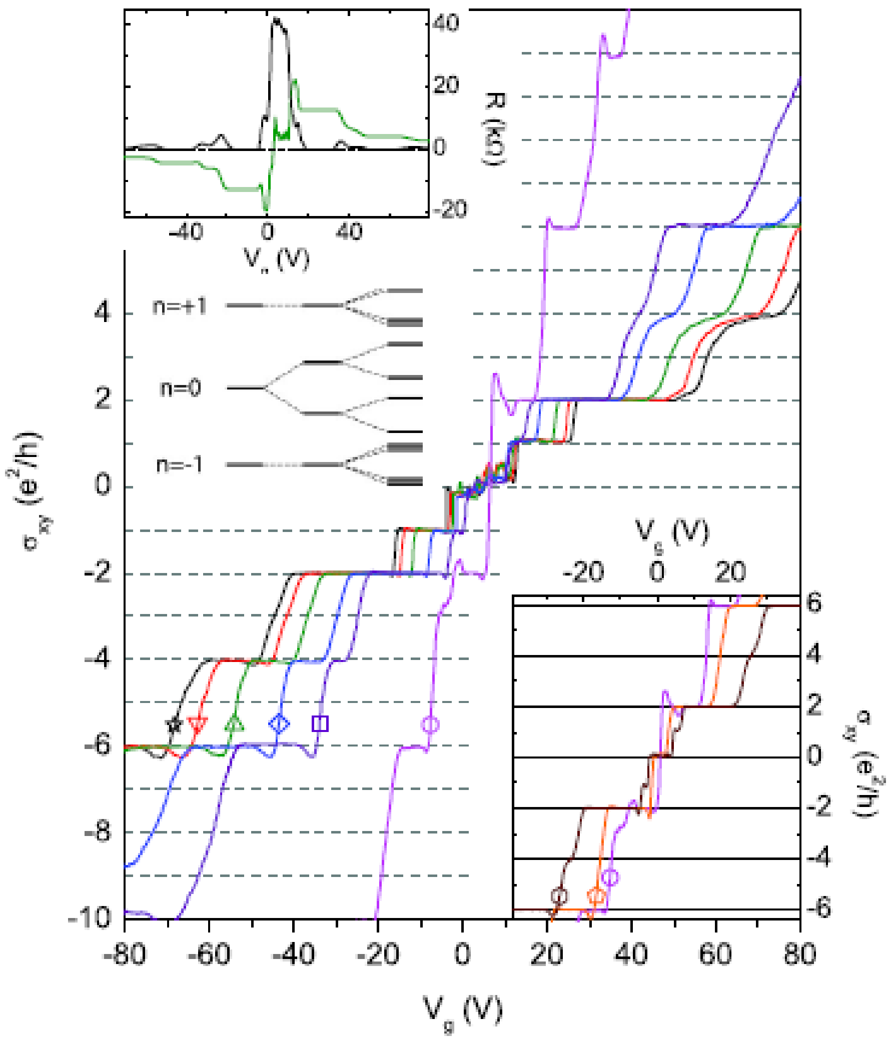
- The spin+valley symmetry of the central LL(n=0) breaks down at higher magnetic field

- Conductance steps at
  -2, +2

- Conductance steps at
  -2, 0, +2

- Conductance steps at
  -2, -1, 0, 1, 2

B



Y. Zhang, et, al., PRL **96,** 136806 (2006)

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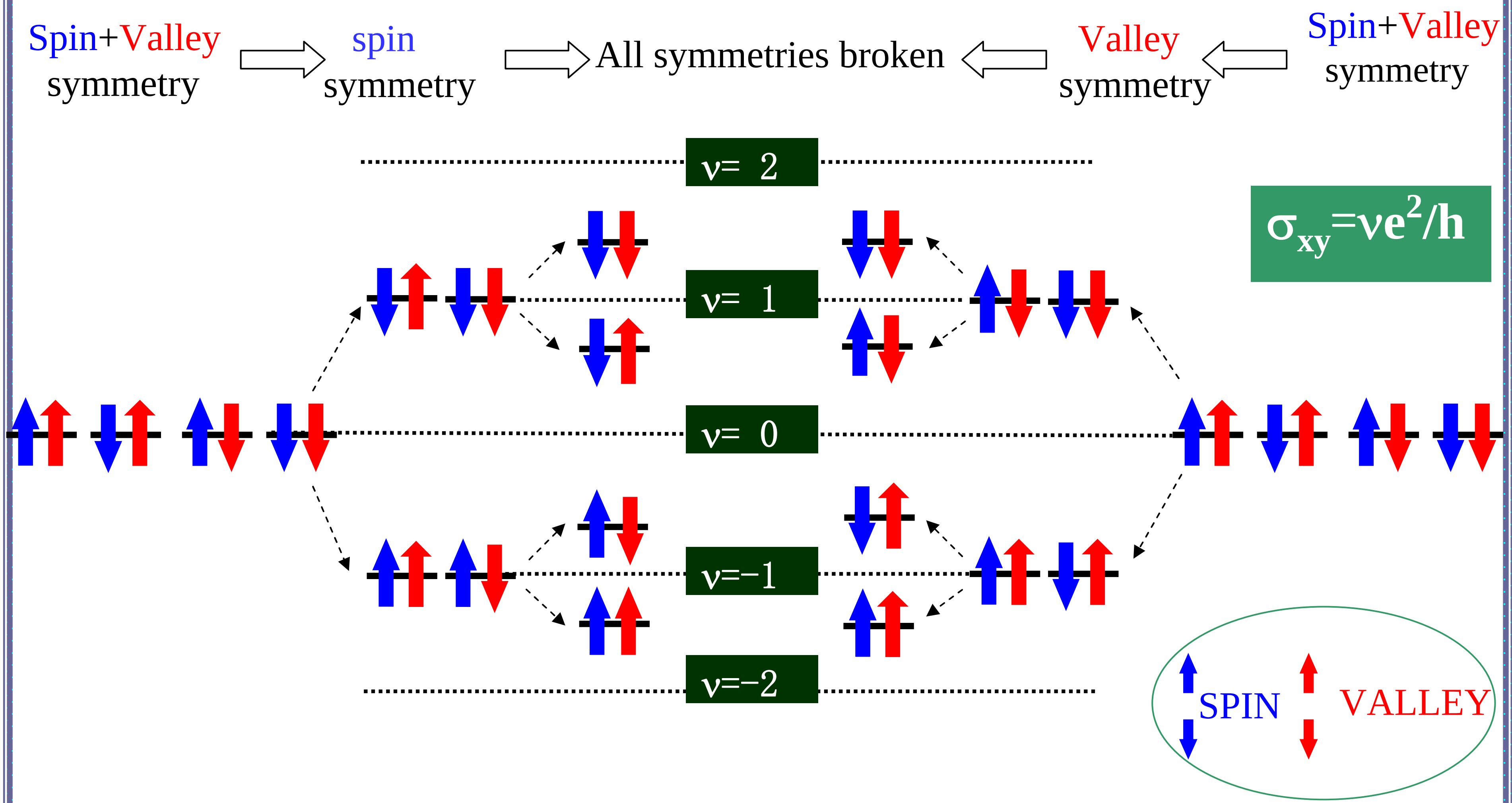
Which symmetry will be broken first, Spin or valley?



J.G.Checkelsky,L. Li,N.P.Ong, PRL 100,2008
PRB 79, 2009
J. Fuchs, P.Lederer, PRL 98, 2007

D.A. Abanin,P.A.Lee,L.S.Levitov,PRL 96,2006
D.A.Abanin,K.S.Novoselov,el.al., PRL 98,2007
Z.Jiang,Y.Zhang,et.al, PRL 99, 2007

Mechanisms of symmetry breaking

- LL n=0 ➔ Kinetic energy is quenched
- Hamiltonian governing LL physics consists of

- Coulomb interaction

$$H^C \sim \frac{1}{2}\int_{rr'} V_{rr'}\rho_{r'}\rho_r \sim H^D + H^{EX}$$

- Coulomb exchange is dominant: H^{EX}

- Zeeman interaction

$$H^B \sim \sum_{\sigma\tau} \sigma B_\sigma \Psi^\dagger_{\sigma\tau}(r)\Psi_{\sigma\tau}(r)$$

Spin: $\uparrow = |\sigma+\rangle$, $\downarrow = |\sigma-\rangle$

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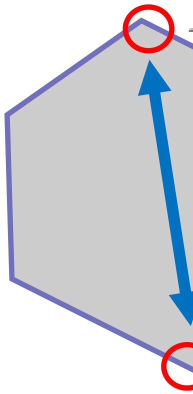
Mechanisms of symmetry breaking

σ=spin index
τ=valley index

- Valley-scattering impurity

$$H^{imp} \sim \sum_{\sigma\tau} V'(r)\Psi^{\dagger}_{\sigma\tau}(r)\Psi_{\sigma\bar{\tau}}(r)$$



- Electron-phonon coupling

$$H^{el-ph} \sim \sum_{\sigma\tau} \lambda u_{2K}\Psi^{\dagger}_{\sigma\tau}(r)\Psi_{\sigma\bar{\tau}}(r)$$

If phonon condenses (Kekule distortion), it will generate effective valley-polarizing field, much like spin-polarizing Zeeman field!

$$\sim \sum_{\sigma\tau} \lambda \langle u_{2K}\rangle \Psi^{+}_{\sigma\tau}\Psi_{\sigma\bar{\tau}} \sim U\sum_{\sigma\tau}\Psi^{+}_{\sigma\tau}\Psi_{\sigma\bar{\tau}}$$

Valley:

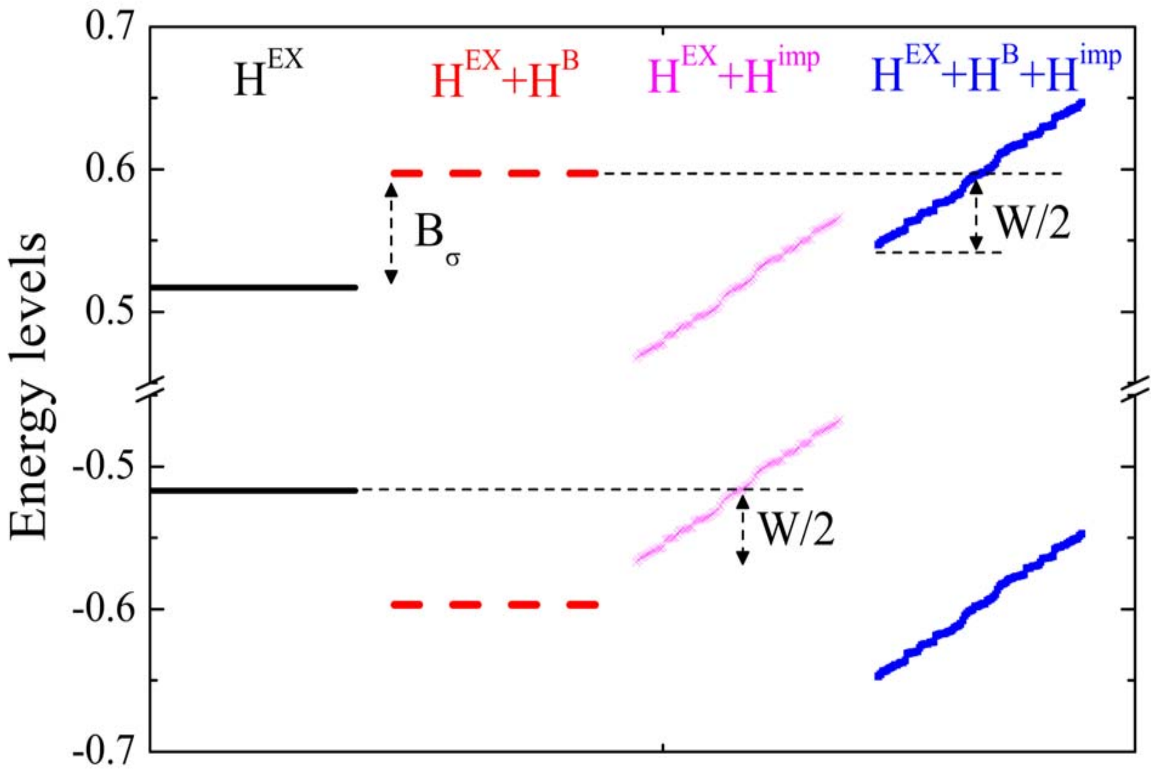
$$\uparrow = \frac{1}{\sqrt{2}}(|\tau=+\rangle + |\tau=-\rangle)$$

$$\downarrow = \frac{1}{\sqrt{2}}(|\tau=+\rangle - |\tau=-\rangle)$$

K. Nomura, S. Ryu, and D.-H. Lee, arXiv:0906.0159v1

Self-consistent Hartree-Fock theory(SCHFT)

- Energy levels at half-filling from self-consistent solutions



- The number of flux through the lattice $N_\phi = 50$
- The impurity broadening W=0.1
- The Zeeman field is B_σ=0.08
- Exchange energy E^{EX}~0.52

- H^{EX} alone, the initial 4-fold degeneracy of the LL is split : $\pm E^{EX}$
- Inclusion of Zeeman field => spin-SU(2) symmetry is broken
- Valley-scattering impurity does not result in additional splitting levels and gives rise to broadened energy levels of width W.

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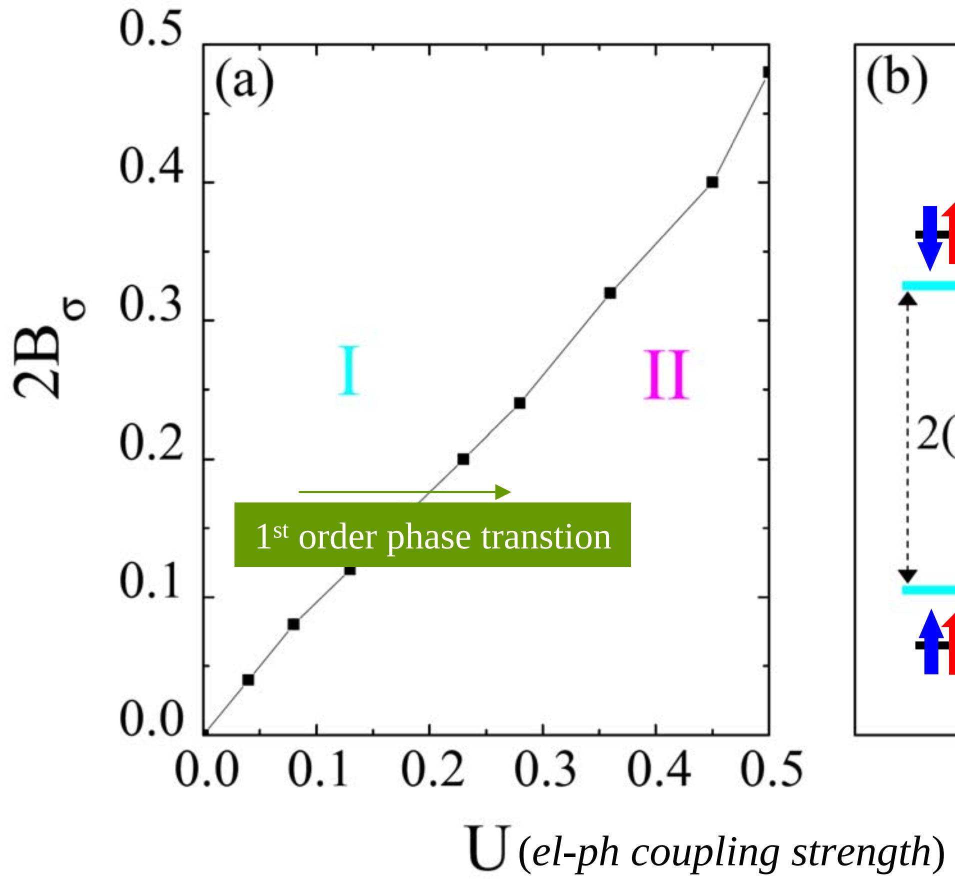
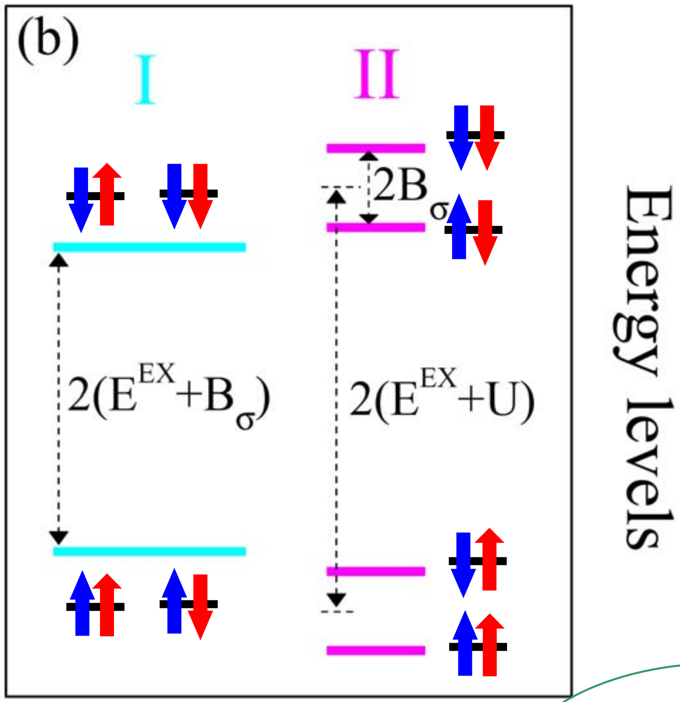
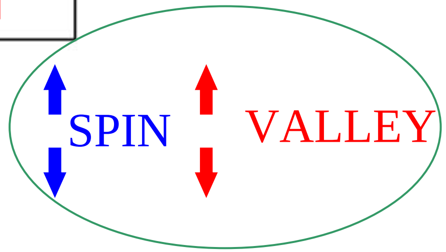
SCHFT: $H^{EX}+H^{el\text{-}ph}+H^{B}$ (half-filling case)

(a) Phase diagram of the CLL(n=0) (b) Schematic energy levels







- In I, Zeeman effect dominates, spin-polarized level splitting
- In II, el-ph interaction U dominates, main polarization direction is along valley axis

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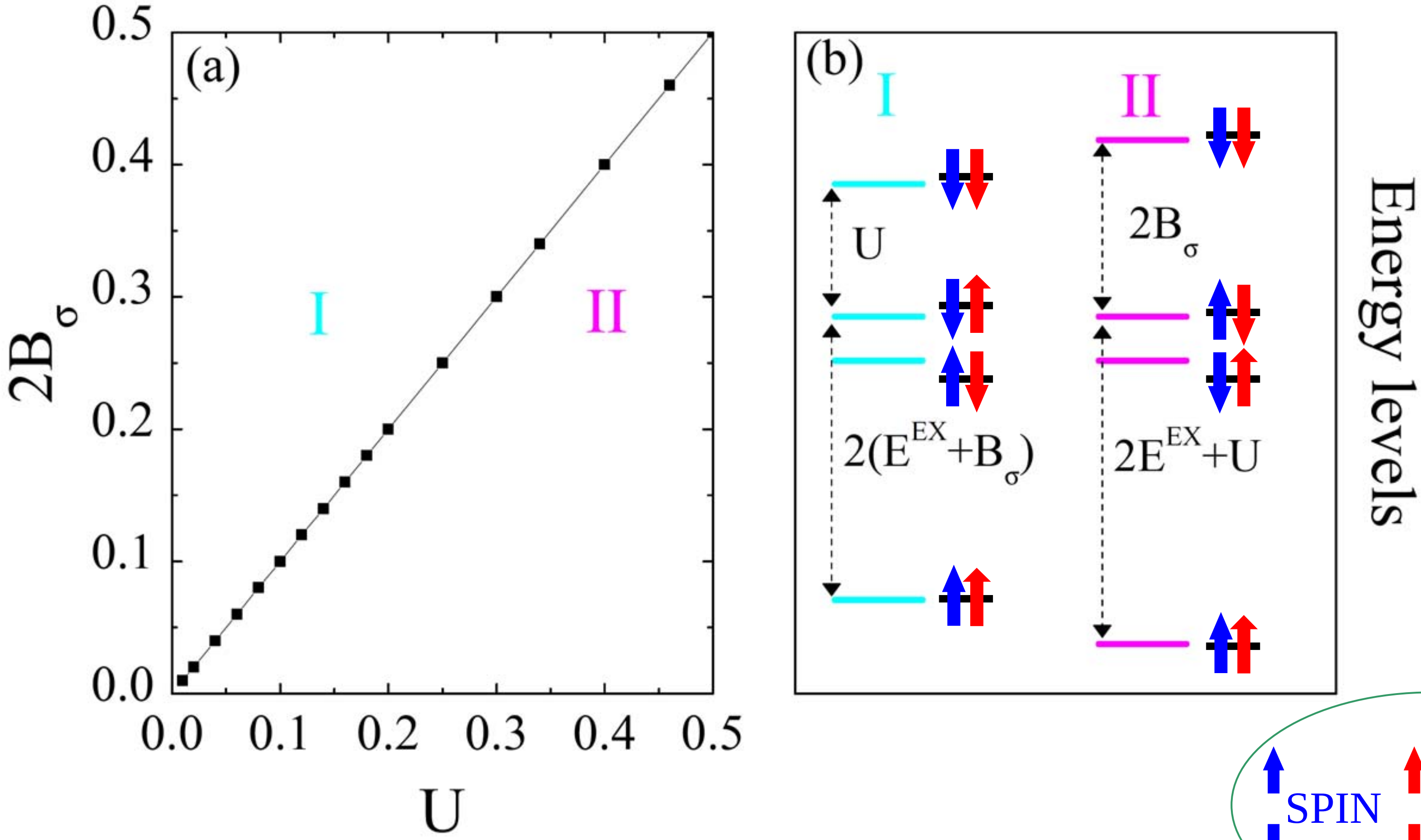
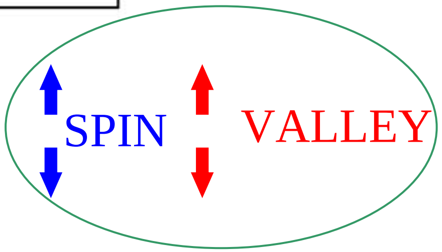
SCHFT: $H^{EX}+H^{el\text{-}ph}+H^{B}$ (quarter-filling case)





"valley-first" –split region I ⟷ "spin first" –split region II

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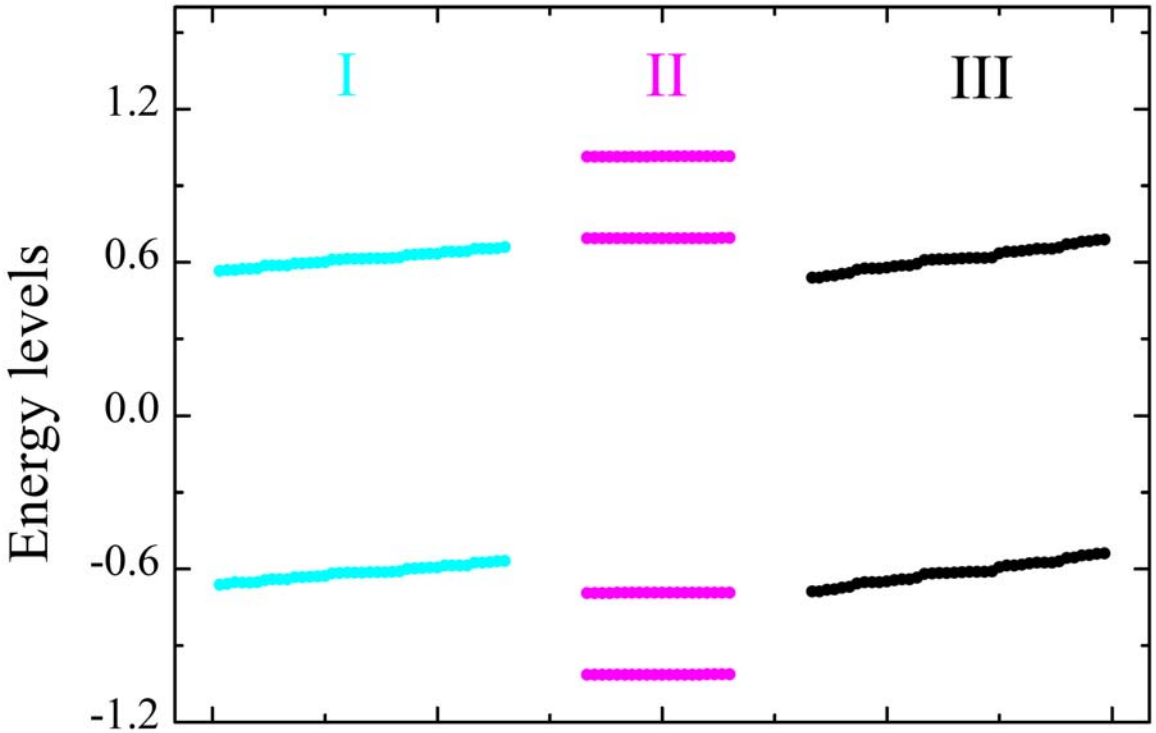
SCHFT: $H^{EX}+H^{el\text{-}ph}+H^{B}+H^{imp}$(half-filling case)

- Schematic energy levels



I: $(2B_\sigma,U)=(0.32,0.36)$, W=0.1

II: $(2B_\sigma,U)=(0.32,0.4)$, W=0.1

III: $(2B_\sigma,U)=(0.32,0.4)$, W=0.15

$(2B_\sigma,U)=(0.32,0.4)$, $W_c=0.12$

- In region I , the impurity does not alter the basic features of phase diagram $(2B_\sigma, U)$,
- In region II, the impurity does not alter the features of phase diagram (2Bs, U) when $W<W_c$,
- When $W>W_c$ the original 4-fold splitting levels in region II disappear => Region III.

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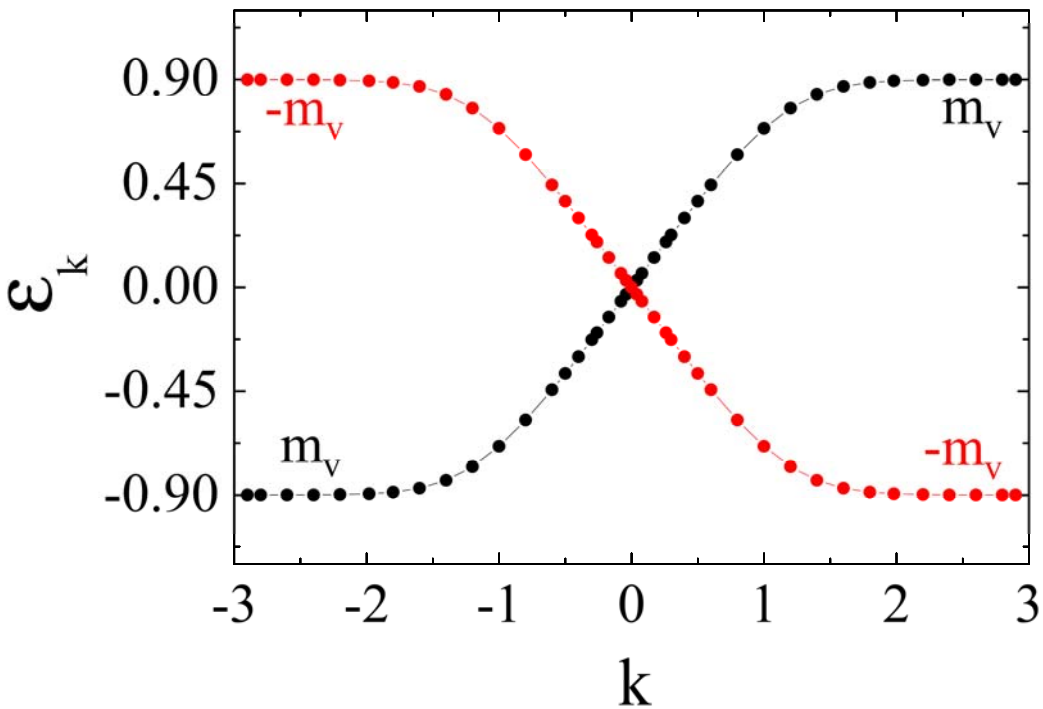
Edges and doman wall

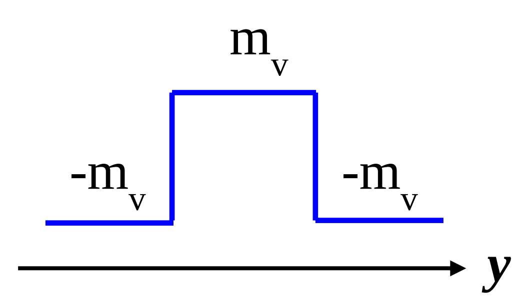
- Valley domain walls may exist because $E(\sigma,\tau) = E(\sigma,\bar{\tau})$ due to Zeeman splitting
- Considering a spinless case first, $H = H^K + H^m$

$$H^K \sim \hbar v_F \sum \Psi_\tau^\dagger \left(k_x\sigma_y + \tau k_y \sigma_x\right)\Psi_\tau, \quad \Psi_\tau = \begin{pmatrix} \Psi_{A\tau} \\ \Psi_{B\tau} \end{pmatrix}$$

$$H^m \sim m(y) \sum_\tau \Psi^\dagger_{A\bar{\tau}} \Psi_{B\tau} + \Psi^\dagger_{B\bar{\tau}}\Psi_{A\tau}$$



$$m(y) = \begin{cases} m_v, & y > 0 \\ -m_v, & y < 0 \end{cases}$$



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Conlusion and Outlook

- Several microscopic mechanisms are explored=>breaking n=0 4-fold LL degeneracy in a single-layer graphene

  Coulomb exchange, Zeeman interaction, valley-scattering impurity, electron-phonon interaction

- The competitive nature of the valley-splitting(el-ph interaction) and spin-splitting(Zeeman interaction) leads to a phase diagram

  Either "spin first" or "valley first" level splitting

- Gapless states exist when LLs with opposite valley polarities form a domain wall

- Our calculation suggests full LL splitting implies CDW order

- Graphene QHE in single and bilayers shows interesting symmetry breaking pattern

  Bilayer case will be more complicated

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