

Chiral Spin Liquid from Dzyaloshinskii-Moriya Interactions

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Introduction to spin liquids

● Spin Liquids

- Retains rotational and translational symmetries down to the lowest temperatures
- No magnetic long range order
- Chiral spin liquids (chiral order present)
 - Scalar chiral spin liquid (sCSL) breaks time-reversal symmetry
→ $\langle \mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k \rangle \neq 0$
 - Vector chiral spin liquid (vCSL) breaks inversion symmetry
→ $\langle \mathbf{z} \cdot \mathbf{S}_i \times \mathbf{S}_j \rangle \neq 0$
- Chirality arises due to frustration from geometry or competing interactions
 - Triangular, kagome, pyrochlore, J_1 - J_2 , *etc.*



Studies on spin liquids

- Major research topic for past couple of decades
 - Search for spin liquid states usually starts with Heisenberg Hamiltonian on various frustrated lattices
 - Kalmeyer & Laughlin, PRL **59**, 2095 (87)
 - Wen, Wilczek, & Zee, PRB **39**, 11413 (89)
 - Systems with larger atomic numbers and complex lattices with less symmetry becoming more important
 - Spotlight on Dzyaloshinskii–Moriya (DM) interaction effects
 - **Na₃Ir₃O₈ (Hyper-Kagome spin liquid)**
Okamoto, Nohara, Aruga-Katori, & Takagi, PRL **99**, 137207 (07)
Chen & Balents, PRB **78**, 094403 (08)
Zhou, Lee, Ng, & Zhang, PRL **101**, 197201 (08)
- **What effect does DM have on spin liquid physics?**



Method of approach

- J_1 - J_2 spin model without DM
 1. Begin with fermionic mean field theory (fMFT) on the J_1 - J_2 Heisenberg Hamiltonian
 2. Map out phase diagram w.r.t. J_2/J_1 (degree of frustration)
 3. Compare the energies of the mean field states using variational Monte Carlo (VMC)
 4. What are the (low-lying) excitations?
 5. Order parameter (chirality: scalar or vector?)
 6. Add DM interactions and repeat steps 1. through 5.

+ DM  New Phase?



Fermionic mean field theory (fMFT)

- J_1 - J_2 spin model on a square lattice

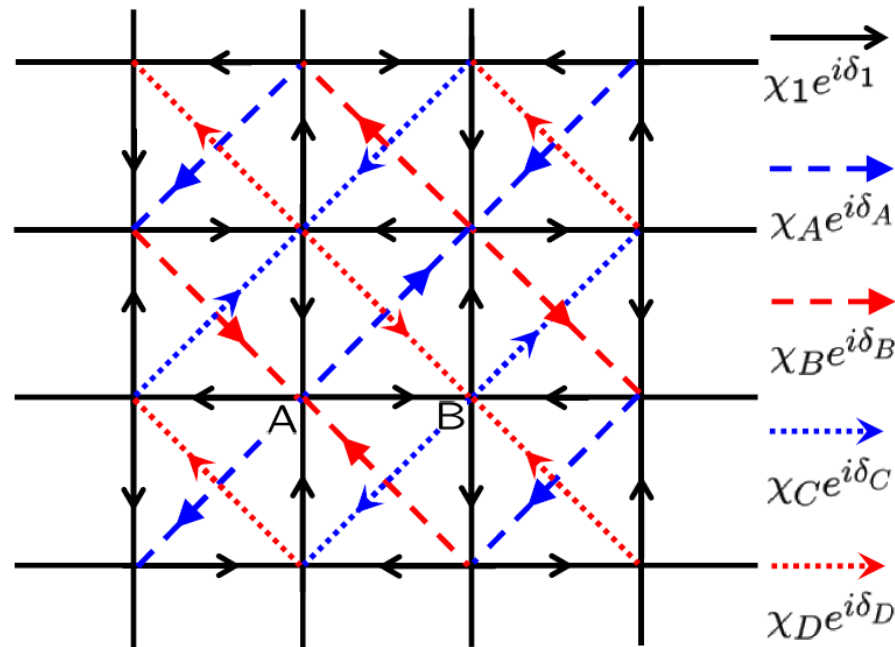
$$H = J_1 \sum_{\langle ij \rangle} S_i \cdot S_j + J_2 \sum_{\langle ik \rangle} S_i \cdot S_k$$

- Constraint : no charge fluctuations, $n_i=1$
- Rewrite spins as fermionic bilinears
- Define $\chi_{ij,\sigma}$: hopping between sites
- Apply mean field theory

$$S_i = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta} \quad \chi_{ij,\sigma} = f_{i\sigma}^\dagger f_{j\sigma}$$

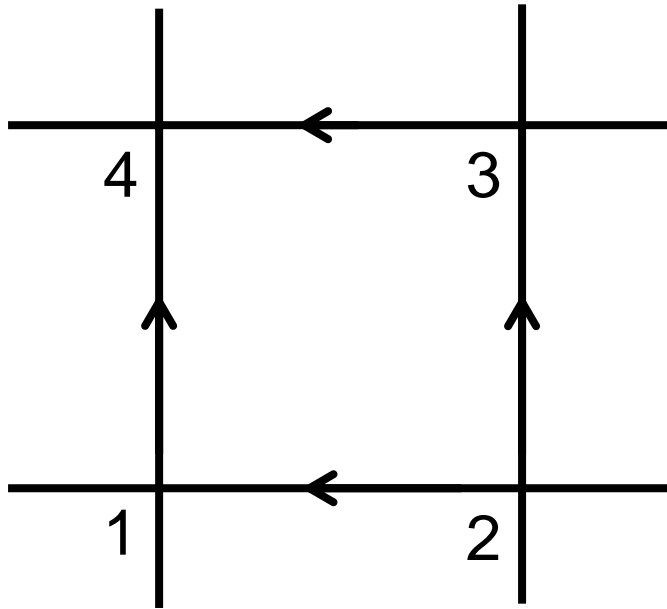


$$H = -J_1 \sum_{\langle ij,\sigma \rangle} \langle \chi_{ij} \rangle f_{j,\sigma}^\dagger f_{i,\sigma} - J_2 \sum_{\langle ik,\sigma \rangle} \langle \chi_{ik} \rangle f_{k,\sigma}^\dagger f_{i,\sigma} + h.c.$$



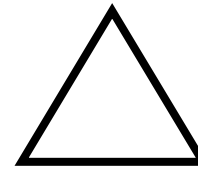
Flux States – Rokhsar's Rule

Rokhsar PRL 65, 1506 (1990)

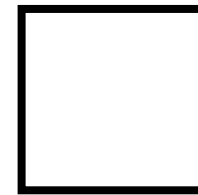


$$\prod \chi_{ij} = \chi_{12}\chi_{23}\chi_{34}\chi_{41} = \chi e^{i\Phi}$$

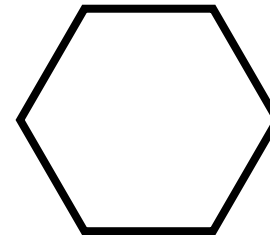
Φ : (physical) flux



$$\Phi = \frac{\pi}{2}$$



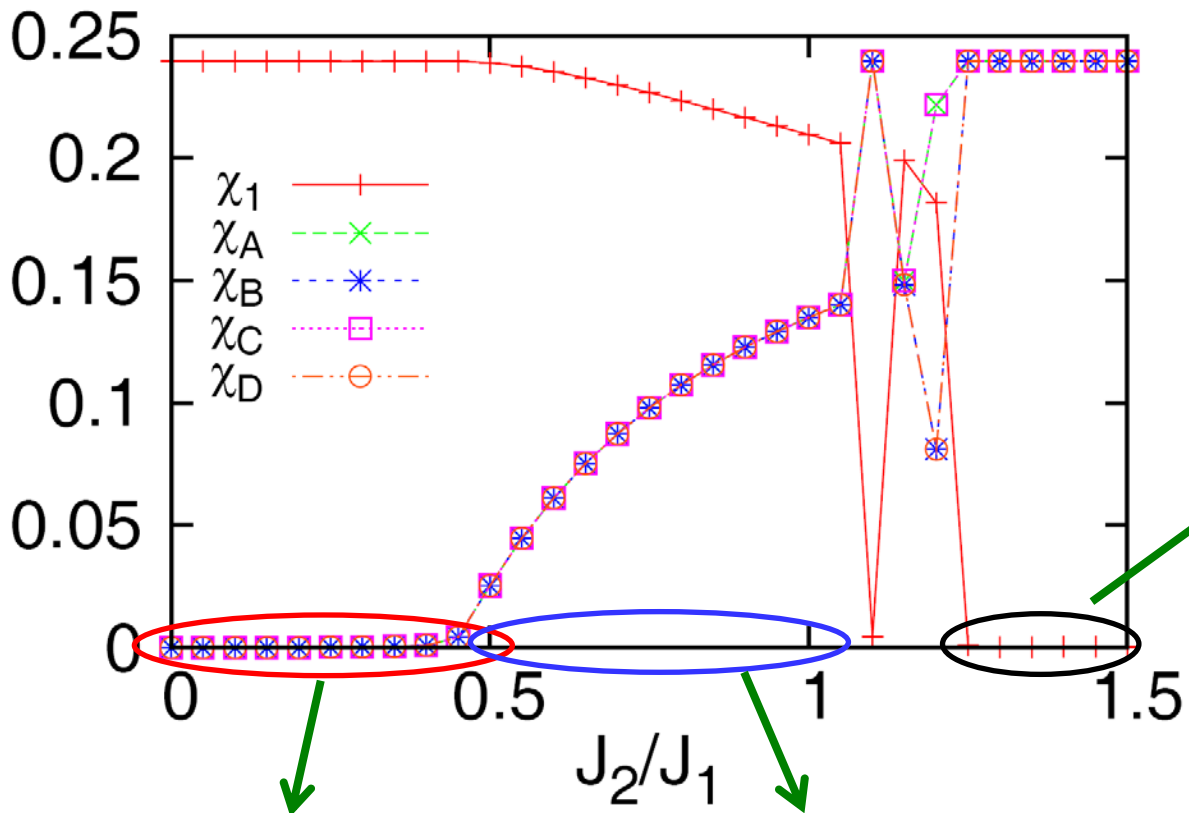
$$\Phi = \pi$$



$$\Phi = 0$$



Phase diagram of J_1 - J_2 without DM



See also :
 Wen, Wilczek, & Zee
 PRB 39, 11413 (89)

Bipartite π -flux
 $\chi_A = \chi_B = \chi_C = \chi_D \neq 0,$
 $\chi_1 = 0$

π -flux

(s)CSL

$$\chi_A = \chi_B = \chi_C = \chi_D = 0,$$

$$\chi_1 \neq 0, \delta_1 = \pi/4$$

$$\chi_A = \chi_B = \chi_C = \chi_D \neq 0,$$

$$\delta_A = \delta_D = 0, \delta_B = \delta_C = \pi,$$

$$\delta_1 = \pi/4$$

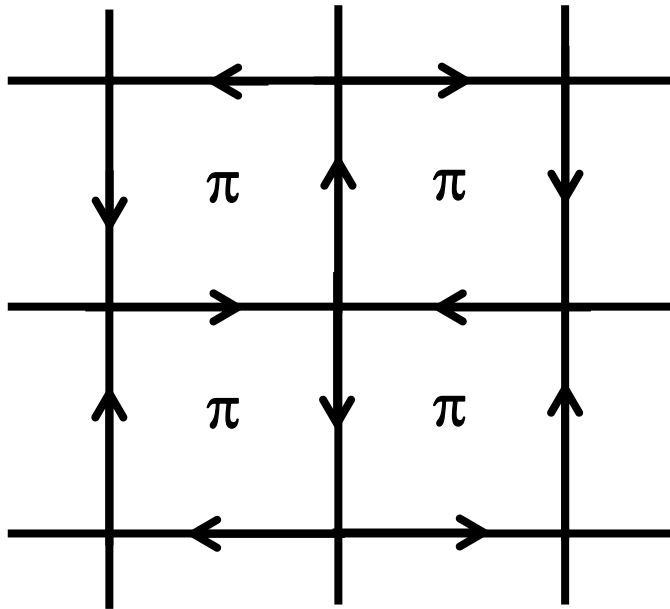


π -flux phases

π -flux

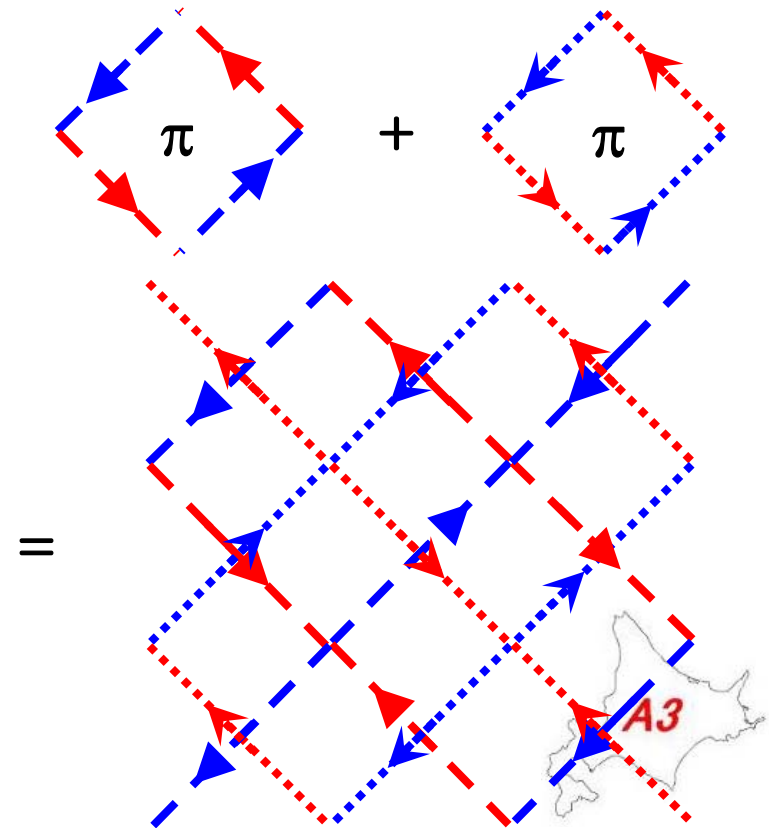
$$\chi_A = \chi_B = \chi_C = \chi_D = 0, \\ \chi_1 \neq 0, \delta_1 = \pi/4$$

$$\longrightarrow \chi_1 e^{i\delta_1}$$



Bipartite π -flux

$$\chi_A = \chi_B = \chi_C = \chi_D \neq 0, \\ \chi_1 = 0$$



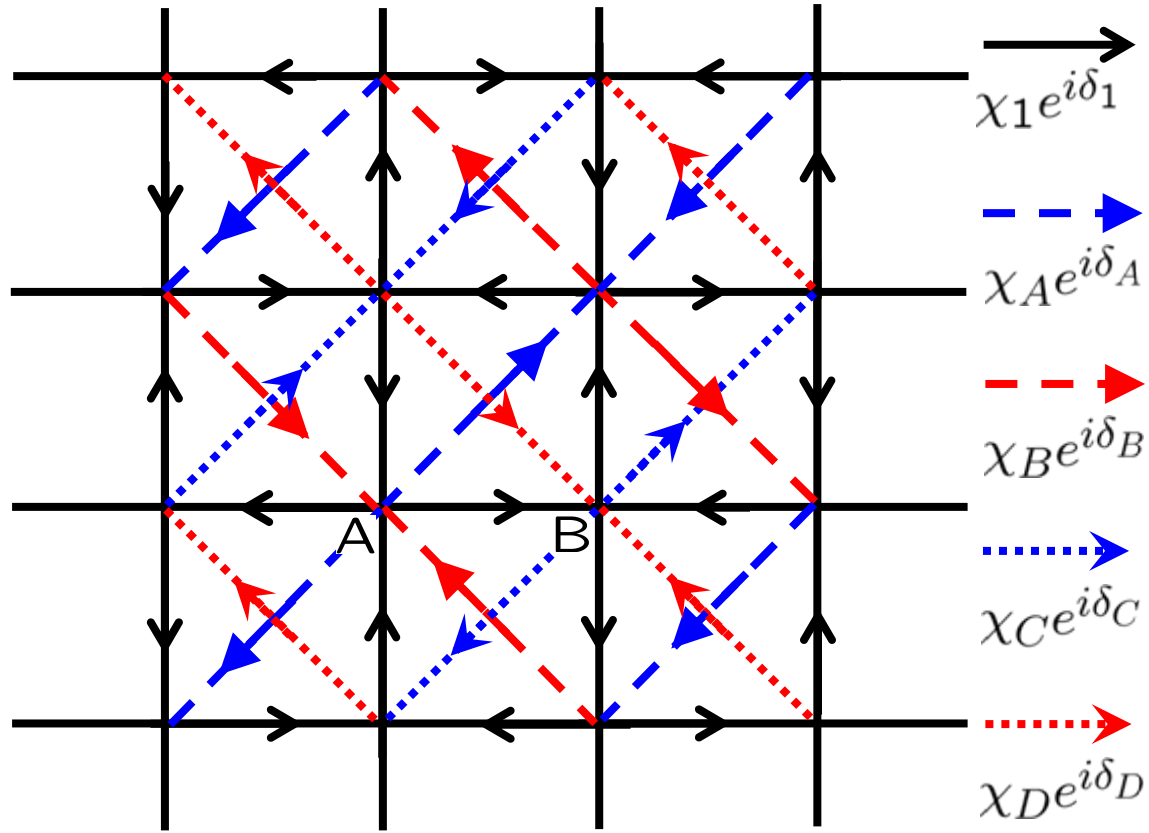
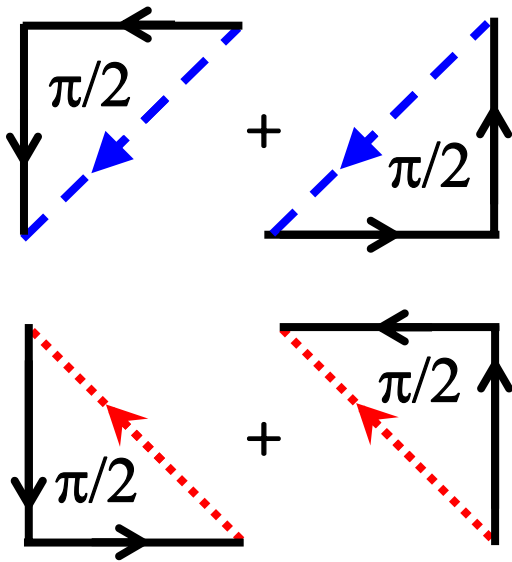
Chiral spin liquid phase

Chiral Spin Liquid

$$\chi_A = \chi_B = \chi_C = \chi_D \neq 0,$$

$$\delta_A = \delta_D = 0, \delta_B = \delta_C = \pi,$$

$$\delta_1 = \pi/4$$

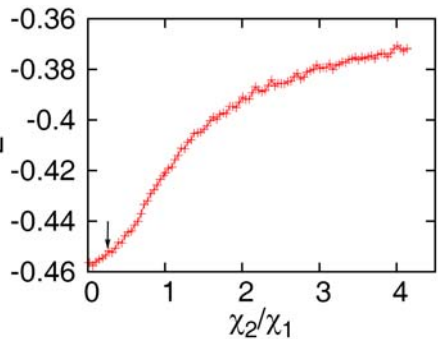


Both up and down spin species
“feels” a flux of $\pi/2$ for each triangle

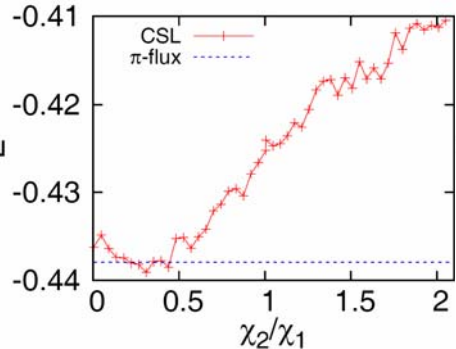


Variational Monte Carlo (VMC) results

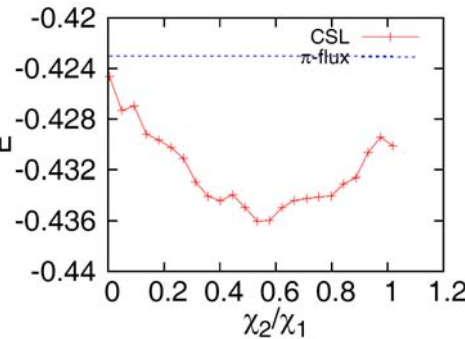
$J_2/J_1=0.6$



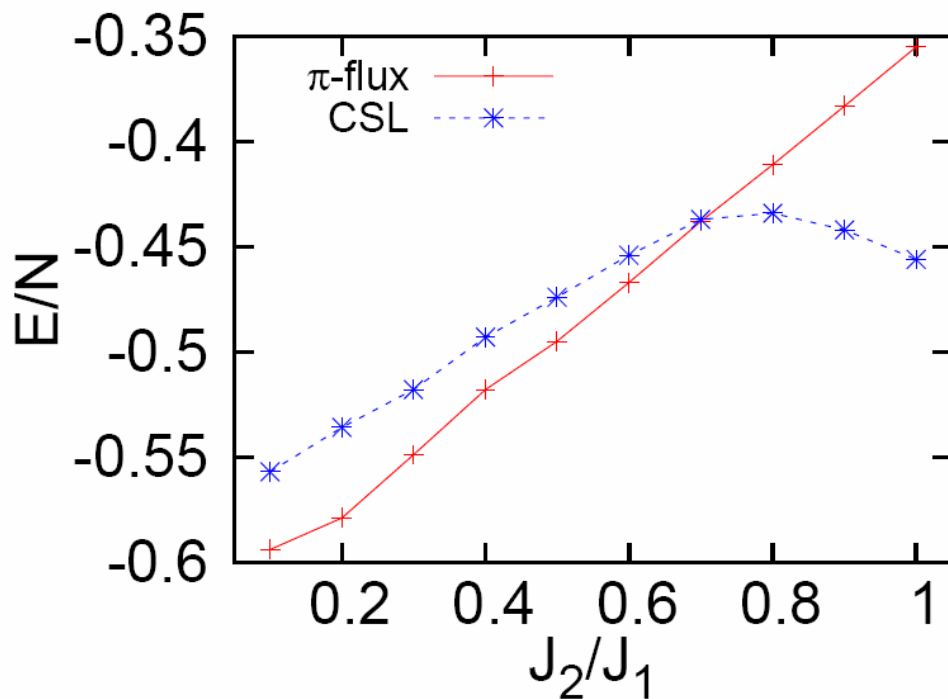
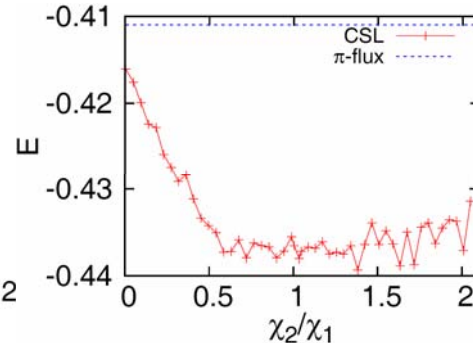
$J_2/J_1=0.7$



$J_2/J_1=0.75$



$J_2/J_1=0.8$



J_2/J_1	π -flux	CSL	χ_2/χ_1
0.70	-0.438	-0.439	0.31
0.75	-0.423	-0.436	0.53
0.80	-0.411	-0.438	1.03*



Heisenberg + DM (HDM model)

- DM interaction in fermion representation

$$(S_i \times S_j)^z = -\frac{i}{2}(\chi_{ij\uparrow}\chi_{ji\downarrow} - \chi_{ij\downarrow}\chi_{ji\uparrow})$$

- H_{HDM} model has the following form

$$\Rightarrow JS_i \cdot S_j + K(S_i \times S_j)^z \sim -J \sum_{\sigma} \chi_{ij\sigma} \chi_{ji\sigma} - J' \sum_{\sigma} e^{iF\sigma} \chi_{ij\sigma} \chi_{ji\bar{\sigma}}$$

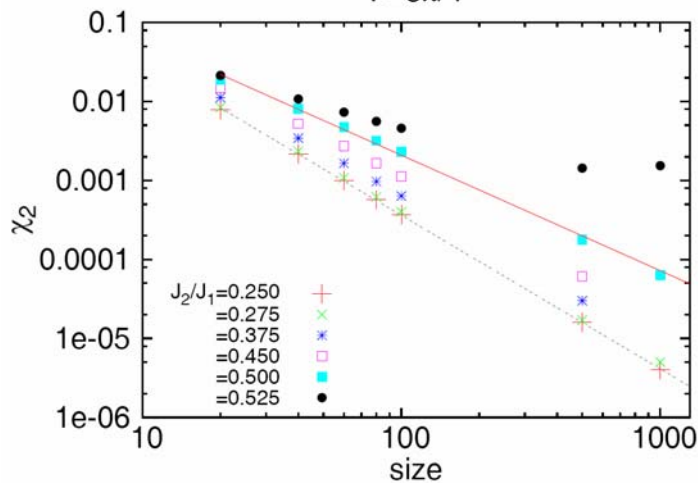
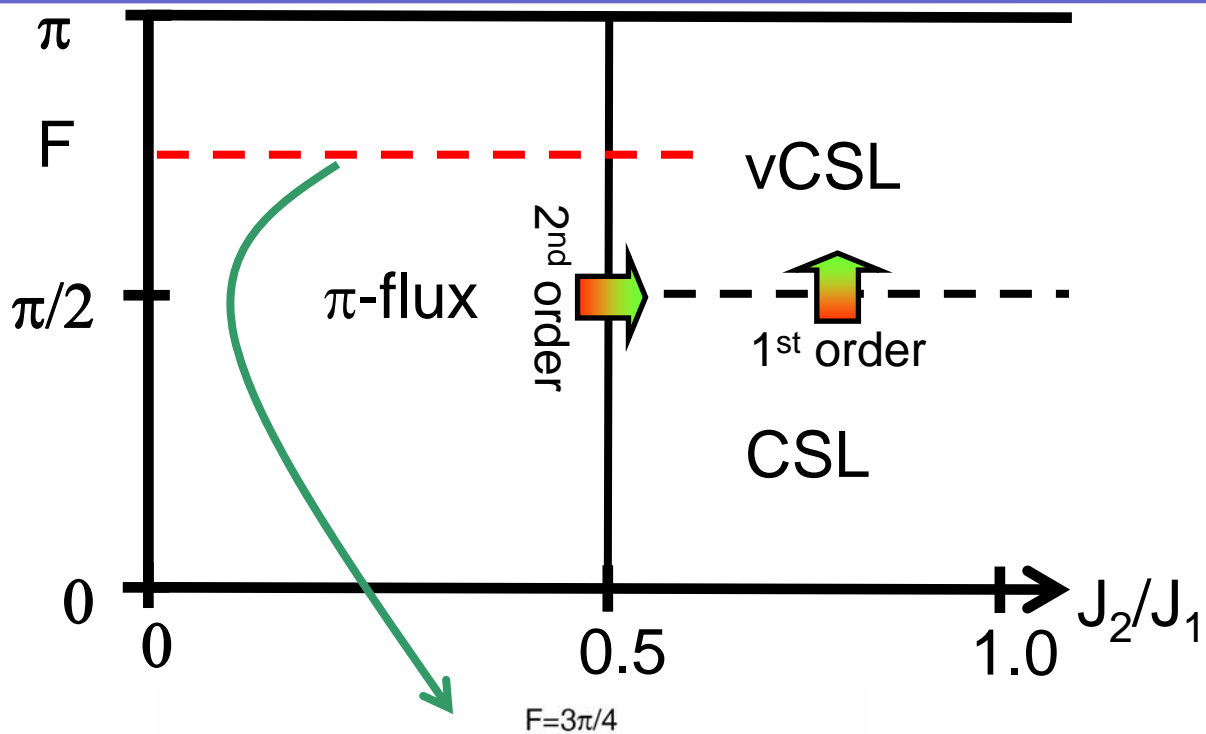
- F can be thought of as the strength of the DM interaction
 - $\tan(F)=K/J$
 - Decouple spin dependent gauge flux $e^{iF\sigma}$ by

$$f_{i,\sigma} \rightarrow e^{iF(x_i+y_i)\sigma/2} f_{i,\sigma}$$

- Introduce NNN coupling to stabilize DM effects



Phase diagram of H_{HDM}



Vector chiral spin liquid (vCSL) phase

vCSL

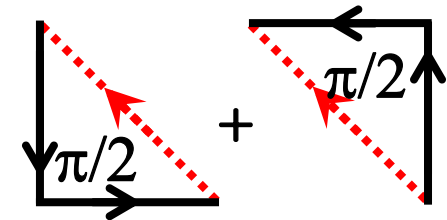
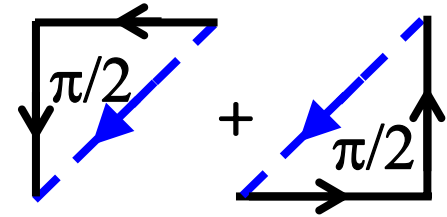
$$\delta_1 = \pi/4, \chi_2 \neq 0,$$

$$\delta_{A,\sigma} = \delta_{B,\bar{\sigma}} = \delta_{C,\bar{\sigma}} = \delta_{D,\sigma} = \pi,$$

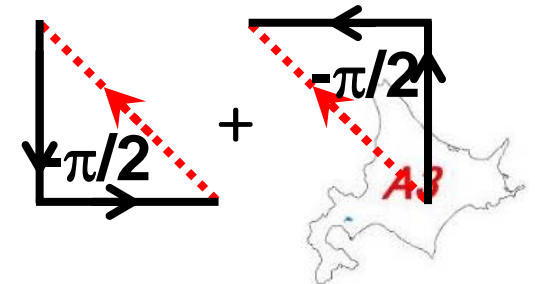
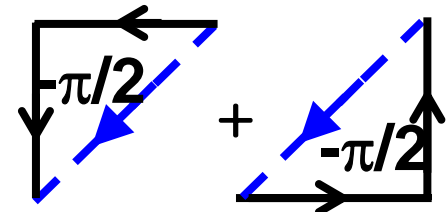
$$\delta_{A,\bar{\sigma}} = \delta_{B,\sigma} = \delta_{C,\sigma} = \delta_{D,\bar{\sigma}} = 0,$$

up spins : flux of $\pi/2$ per triangle
down spins : flux of $-\pi/2$ per triangle

Up spins :



Down spins :



Conclusions & future work

- The introduction of DM leads to a new phase, a vector chiral spin phase.
- vCSL phase can be reached from a CSL phase by increasing the DM strength past a certain critical value (first order phase transition).
- In the vCSL phase, the transverse spin Hall conductivity is no longer zero.
- Ground state of H_{HDM} has been established. What about spinon excitations?
- Through VMC, can calculate the vector chirality of vCSL phase. Is it really non-zero?

